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Guessing with negative feedback: An experiment

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ABSTRACT

We investigate experimentally a new variant of the beauty contest game (BCG) in which players' actions are strategic substitutes (a negative feedback BCG). Our results show that chosen numbers are closer to the rational expectation equilibrium than in a strategic complements environment (a positive feedback BCG). We also find that the estimated average depth of reasoning from the cognitive hierarchy model does not differ between the two environments. We show that the difference may be attributed to the fact that additional information is more valuable when players' actions are strategic substitutes rather than strategic complements, in line with other recent experimental findings.

1. Introduction

Most speculative markets are driven by future price expectations. Traders who try to buy (sell) at a low price (high price) need to forecast the time when the excess supply turns to excess demand and conversely. To make such a guess, each trader has to guess not only the other traders' excess demand forecasts, but also other traders guesses about other traders' forecasts, ad infinitum. Fundamentally all traders' expectations are interdependent. Under common knowledge of rationality, the guessing game boils down to a fixed point solution where all traders hold the same expectations. However, if the assumption of common knowledge of rationality is relaxed, thus acknowledging for heterogeneity in guessing abilities, the outcome becomes highly unpredictable.

Beauty contest games (BCG) provide an attractive framework that yields insights into how subjects make guesses about other subjects' expectations in a laboratory setting. BCG have two interesting features that facilitate understanding depth of reasoning: first, they have a unique solution (under suitable restrictions), and second, BCG games are based on simple guessing rules, i.e. iterated elimination of dominated strategies through eductive[1] reasoning (Binmore, 1987, 1988; Guesnerie, 1992). The standard BCG assumes that M players have to choose simultaneously a number from the closed interval (0, 100), the winner being the player whose chosen number is closest to p times the mean, with $p \in (0,1)$. The winner is entitled to a fixed prize, which is split equally in case of ties.

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[1] Following Binmore (1987), the word *eductive* (to educe = to bring, to drow out, develop, extract or evolve from latent of potential existence; infer a number, a principle, from data or from another state in which it previously existed, from the Latin word *educere*, lead; Oxford English Dictionary) is used to describe a dynamic process by means of which equilibrium is achieved through careful reasoning on the part of the players before and during the play of the game.

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A number of papers have conducted experiments based on BCG. One finding is that in a population of equally well-informed subjects, average numbers are far from the predicted winning number (Nagel, 1995, 1998; Camerer, 2003). Furthermore subjects seem to perform only a few steps of reasoning (about 2) and have heterogeneous guessing abilities. Several models have been proposed to capture individual differences in guessing ability or stressed popular reasons to choose a particular number (see Stahl, 1996, 1998; Camerer et al., 2004; Guth et al., 2002).[2]

Some other recent experimental findings about price guessing games (Fehr and Tyran, 2008) demonstrated that the strategic environment matters. In their paper, the authors examined this question in the context of the adjustment of nominal prices after an anticipated (exogenous) monetary shock, and showed that when agents' actions are strategic substitutes, adjustment to the new equilibrium is extremely quick, whereas under strategic complementarity, adjustment is both very slow and associated with relatively large real effects. These findings support the predictions of Haltiwanger and Waldman (1985, 1989), who showed that in a heterogeneous population composed of rational and irrational agents, the speed of adjustment to the equilibrium price depends on whether agent's actions are strategic complements or substitutes. When individual actions are strategic substitutes, irrational behaviour has less influence on the adjustment process than if actions are strategic complements. The intuition is that "the presence of strategic complements causes the sophisticated to have a rational incentive to imitate the less wise in equilibrium" (Haltiwanger and Waldman, 1989). Such incentive for sophisticated agents to imitate naïve agents exacerbates the adjustment bias initiated by naïve agents.

Fehr and Tyran (2008) found evidence for the prediction of Haltiwanger and Waldman (1985, 1989) in their laboratory experiment with large exogenous shocks, whereas, in another experiment, Heemeijer et al. (2008) observed that the adjustment speed towards the equilibrium is faster under strategic substitutes (negative feedback) than under strategic complements (positive feedback) even if the market price is perturbed only by small random (zero mean) fluctuations in each period. In their experiment, subjects had to guess the next period market price, defined as a linear function of the average guess. Subjects were only informed about past realized market prices and their own past price expectations, but were unaware of the pricing rule. Therefore, it is likely that the strategic environment, i.e. the type of feedback rule, affects subjects' expectation rule even in a nearly stable environment. They suggest that incentives to adopt contrarian behaviour under negative feedback tend to destroy the tendency of trend-following behaviour observed under positive feedback. Therefore, we conjecture that the type of feedback influences the way subjects form their expectations even in the absence of exogenous shocks.

The aim of our experiment is to investigate the above conjecture in a stable environment (without shocks) where subjects have to perform a simple guessing task with an explicitly known feedback rule. To do this, we compare experimentally two variants of the standard BCG with the same unique and interior solution. In each game the subjects' task is to choose a number from the same interval. In the positive feedback game, the winner is the player choosing the closest number to $p \times (mean+c)$, where p and c are parameters known to all players (with $p \in (0,1)$). In the negative feedback game, the winner is the player choosing the closest number to $h - p \times mean$, where h is another parameter known to all players. In both cases the winner receives a fixed prize, which is eventually split equally in case of ties. While eductive reasoning predicts the same equilibrium for the two games, the underlying process of iterated elimination of dominated strategies differs for negative and positive feedback.

We therefore compare in this paper positive and negative feedback rules in a BCG with a unique interior equilibrium. To our knowledge this is the first experiment on negative feedback in BCG. Under positive feedback players' chosen numbers are strategic complements while under negative feedback the chosen numbers are strategic substitutes. Assuming eductive reasoning, the negative feedback rule generates a convergence process by which weakly dominated strategy intervals are deleted on both sides of the equilibrium point. The process alternates between elimination of low and high numbers until the equilibrium is reached, following an oscillatory pattern. In contrast, under positive feedback weakly dominated strategy intervals are deleted on one side of the equilibrium point generating a monotonic convergence process. We found that numbers are closer to the equilibrium point under negative feedback than under positive feedback in a one-shot experiment. The negative feedback rule seems to allow a more accurate location of the equilibrium point by inexperienced subjects. However, our estimates of the average depth of reasoning, based on the cognitive hierarchy (CH) model, reject the hypothesis of a deeper reasoning under negative feedback. While this model is based on the assumption that players choose numbers which best reply to their estimated distribution of reasoning depths in the population, there might be other reasons why subjects perform more steps of reasoning or better under negative feedback. Our explanation is inspired by the "directed cognition theory" (Gabaix et al., 2006). Subjects adopt a step by step reasoning, comparing the expected value of an additional step to the additional cost of thinking. Due to the alternating elimination process under negative feedback, each step provides more valuable information, compared to positive feedback.

The remainder of this paper is organized as follows. In Section 2 we present the positive and negative feedback BCG and discuss their properties. In Section 3 we present our experiment and in Section 4 our main findings. Section 5 provides a general discussion on our findings, and Section 6 concludes.

[2] In particular the cognitive hierarchy model (Camerer et al. (2004) assumes that each player holds beliefs about other players' reasoning depth, and chooses a number which is the best reply for his estimated distribution of reasoning depths. While the cognitive hierarchy model fits the data of experimental beauty contest games with corner solutions (i.e. zero) quite well there might be other reasons for observing large numbers. For example, subjects might be reluctant to choose extreme end-points from the set of possible numbers. For instance, Guth et al. (2002), found that chosen numbers get closer to the predicted winning number when the game admits an interior equilibrium solution.

2. Positive and negative feedback in BCG

Assume that M players have to choose simultaneously a number from a closed interval (l, h), where l, h are known parameters. We consider two types of rules for selecting the winner of the game: positive and negative feedback. The winner is awarded a fixed prize, which is split equally among winners if there are several. Under the positive feedback rule, the winner is the player whose chosen number is closest to:

$$P \times (mean + c), \tag{1}$$

where *mean* stands for the mean of all chosen numbers, p and c are parameters known to all players with $p \in (0,1)$. This setting describes the BCG with an interior equilibrium (presented for instance in Guth et al., 2002) and will be designated BCG+ thereafter.[3] Under the negative feedback rule, the winner of the game is the player who chooses the number closest to:

$$h - p \times mean. \tag{2}$$

This game will be designated BCG− thereafter.

Under the assumptions of common knowledge of rationality and common knowledge of the rules of the game, iterated elimination of dominated strategies leads to an interior solution, corresponding to the rational expectations equilibrium (REE), equal to:

$$pc/(1-p) \tag{3}$$

in the BCG+, and to

$$h/(1+p) \tag{4}$$

in the BCG−. At the equilibrium, the prize is equally split among all players, each one making a negligible profit if the population is large enough to avoid strategic manipulation.

As we illustrate bellow, in BCG+, iterated elimination of dominated strategies is one-sided with respect to the equilibrium point, while it is two-sided in BCG−. More precisely, elimination of dominated intervals alternates around the equilibrium point in BCG−. Therefore, in the BCG−, convergence to the equilibrium point oscillates, whereas in BCG+ it is monotonic. This mathematical property will help us showing that non-monotonic elimination of dominated strategies help subjects to make more accurate choices.

Under eductive reasoning the process is usually assumed to start at one of the end-points of the strategy interval. However, under weaker behavioural assumptions the process may start as any point x of the strategy interval, e.g. the mid-point. The choice of a different starting point does not alter the qualitative properties of the elimination process. It remains monotonic in BCG+ and oscillating in BCG−. Consider BCG+: in any step of the reasoning process, a player who for instance guesses a high mean chooses a high number, and a player who guesses a low mean chooses a low number, according to the rule stated in Eq. (2). The iterated elimination process is therefore one-sided from the equilibrium. The process is described in expression (5). Starting from some initial point x in the strategy interval (including boundaries), numbers larger than the values indicated by (5) are iteratively eliminated by eductive reasoning:

$$p(x+c), p^2(x+c)+pc, \ldots, p^n(x+c)+pc\frac{1-p^{n-1}}{1-p}. \tag{5}$$

The standard sequence of elimination assuming eductive reasoning is obtained by replacing x by high and low boundaries ("from the top" or "from the bottom").

In contrast, in BCG− elimination occurs on both-sides of the equilibrium. In the process of elimination of dominated strategies, the equilibrium point is reached by alternately eliminating low and then high numbers, starting from an initial value x. In the first step, numbers smaller than $h-px$ are eliminated, in step 2 numbers larger than $h-p(h-px) = h-ph+p^2x$) are eliminated, in step 3, numbers smaller than $h-p(h-ph+p^2x)$ are eliminated and so on. The sequence of bounds generated by the eductive reasoning in this game is described in

$$h-px, h-p(h-px), \ldots, h\frac{1-(-1)^n p^n}{1+p}+(-1)^n p^n x, \ldots . \tag{6}$$

Thanks to our restrictions on the values of p, c, h, and l and to the isomorphism between BCG+ and BCG−, there is a strict correspondence of odd and even bounds in the sequences described by (5) and (6). More precisely, the values of the odd bounds in (5) are equal to the values of the odd bounds in (6). Symmetrically, the even bounds generated by the complementary values to x with respect to h+pc in (5) are equal to the even bounds in (6) (see Fig. 1 for visual details). In other words, in BCG+ a player who follows the sequence described by (5) and starts the process from some initial value x, would iteratively eliminate exactly the same intervals as under negative feedback, provided that he takes into account both the sequence starting with x and the sequence starting with the complementary value of x (with respect to h+pc). Note that in BCG+ a player needs to combine the bounds of two different sequences to eliminate the same intervals as in BCG−.

[3] Note that $c = 0$ corresponds to the standard BCG studied for example by Nagel (1995).

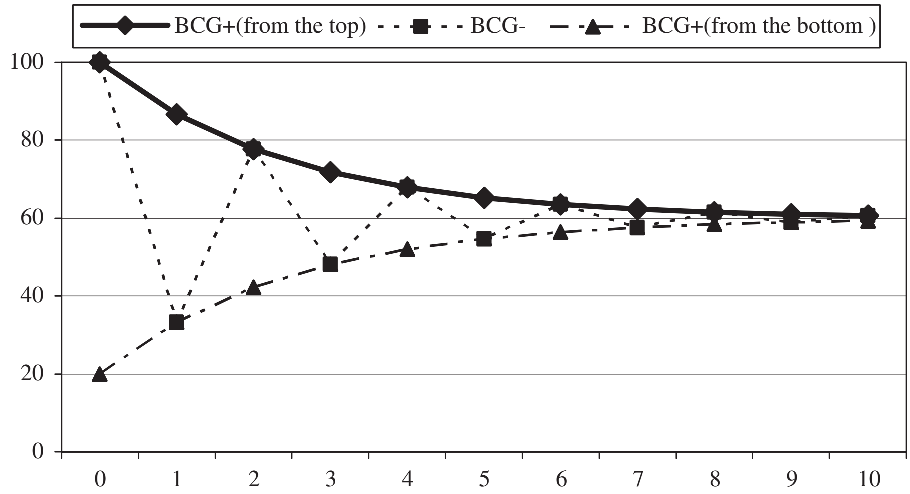

Fig. 1. Eductive reasoning in the BCG−(winning number as a function of the depth of reasoning and the value of p) and in the BCG− and BCG+ for $p = 2/3$.

Table 1
Experimental design.

Type of feedback	Treatment	Definition of mean	Target value	Rational expectation equilibrium	Number of groups
Negative	BCG−	Standard	100−2/3 mean	60	9
Negative	BCG−others	Non-strategic	100−2/3(others' mean)	60	22
Positive	BCG+	Standard	2/3 × (mean+30)	60	4
Positive	BCG+others	Non-strategic	2/3 × (other's mean+30)	60	20

In our experiment we adopted standard parametric boundaries $l = 0$ and $h = 100$. Setting $c = 30$ and $p = 2/3$ leads to the same interior equilibrium, 60, under positive and negative feedback. Since $p < 1$ there is a unique and stable REE, which is the limit value when $n \to \infty$ of the sequences described in (5)–(6). Fig. 1 provides a graphical representation of the eductive process in the BCG− and BCG+ and shows the winning number for iteration steps from 1 to 10 ("from the bottom" and "from the top" boundaries) for $p = 2/3$.

3. Experimental design

Four hundred and forty subjects participated in a one-shot experiment. They were split into 55 independent groups of eight subjects each. The winner in a group received a prize of 8 Euros. In the case of ties, the prize was shared equally among the winners. Participants were randomly assigned either to a BCG− group or to a BCG+ group. Sessions were organized in different locations[4] between May 2004 and October 2007. Participants were students from various disciplines. The software of the computerized experiment was developed within z-Tree (Fischbacher, 2007). On average a session lasted for about 30 minutes overall.

Subjects received written instructions (Appendix A.1). A written questionnaire was submitted to check their understanding before the beginning of the session. Subjects' task was to choose a real number between 0 and 100. In order to control for strategic choices, we implemented a 2×2 factorial design: (positive vs. negative feedback) × (mean number vs. others' mean number). In the "others's mean" treatments, the mean for subject i was defined as the average of all numbers chosen by the other members of his group. It was pointed out to the subjects that their own chosen number had no influence upon the mean. Since groups were relatively small (eight players) it was important to check for possible beliefs players had about one's influence on the average number. Furthermore, in the reference "mean number" treatments, the slopes of the best reply functions differed for the two games: the absolute value for the slope is 7/11 in the BCG−game and 7/13 in the BCG+game (Appendix A.2). This strategic difference is eliminated in the "others' mean number" treatments where the absolute value of the best reply function is the same for both games. Whereas the best-reply functions differ between "mean" treatments and "others' mean" treatments, the equilibrium predictions are the same under both conditions. Table 1 summarizes the experimental design.

[4] Participating laboratories were LEEM (Montpellier), LEES (Strasbourg) and LESSAC (Dijon) all located in France.

Recall that under our parametric restrictions ($l = 0$, $h = 100$, $p = \frac{2}{3}$ and $c = 30$), there is a unique rational and stable expectations equilibrium equal to 60 for both feedback rules. In the result section we keep the notation BCG− and BCG+ for the treatments taking into account the mean of all numbers, while BCG−others and BCG+others identify treatments for which subjects had to guess the mean of the numbers chosen by other players in their group.

4. Results

As summarized in Table 1, we collected data from nine independent groups for BCG− and four independent groups for BCG+. For the non-strategic treatments, we collected data from 22 independent groups for BCG−others and 20 independent groups for BCG+others.

We start with a comparison of the numbers chosen in the strategic and non-strategic treatments.

Result 1. Under negative feedback, chosen numbers are closer to the equilibrium than under positive feedback. Furthermore, numbers chosen in "non-strategic" treatments are closer the equilibrium than in strategic treatments.

Support for Result 1: Fig. 2 shows the frequency distributions of the chosen numbers for our four treatments. Visual inspection of the distributions shows that numbers are closer to the equilibrium point for non-strategic treatments (BCG+others and BCG−others) compared to the standard treatments (BCG+ and BCG−). Furthermore, numbers in negative feedback treatments get closer to the equilibrium point than do those in positive feedback treatments. From Table 2 one can see that the average winning numbers in BCG− and BCG−others are closer to the REE than they are in BCG+ and BCG+others, respectively. We test whether absolute deviations from the REE are smaller in BCG− (BCG−others) than in BCG+ (BCG+others) and find no support for the null hypothesis (p value<0.0076). The percentages of choices at the REE (60) were significantly larger for BCG− and BCG−others than for BCG+ and BCG+others, respectively (p *value*<0.0007).

Result 2. The average depth of reasoning is about two steps in the BCG+ and the BCG− games. While the distribution of the depths of reasoning does not differ across feedback rules, it differs according to the definition of the mean.

Support for Result 2: To test Result 2 we need to assume that a non-negligible fraction of subjects actually rely on models of step-level thinking. While this might not be the case, it is nevertheless important to investigate whether the CH model leads to any difference in depth of reasoning between the two feedback rules. Recall that the CH (Camerer et al., 2004) assumes heterogeneity of depths of reasoning within the population of players (see Appendix A.3): a player who is able to perform k steps of reasoning believes that other players perform at most $k-1$ steps of reasoning. Each player therefore

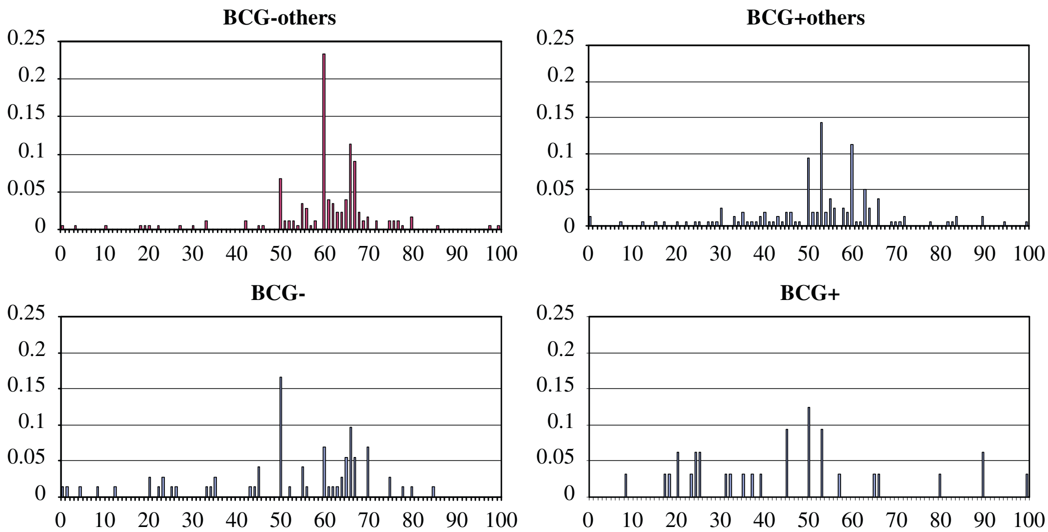

Fig. 2. Choice frequencies per treatment.

Table 2
Depths of reasoning (in steps) and average winning numbers.

Type of feedback	Treatment	Average winning number	Rational expectation equilibrium	Depth of reasoning
Negative	BCG−	56.46	60	1.55
Negative	BCG−others	59.71	60	2.20
Positive	BCG+	43.26	60	1.48
Positive	BCG+others	51.39	60	2.56

Table 3
Distribution of depths of reasoning of the population.

Type of feedback	Treatment	% Of 0-step players	% Of 1-step players	% Of 2-steps players	% Of 3-steps players
Negative	BCG−	27	41	32	0
Negative	BCG−others	14	30	33	24
Positive	BCG+	28	41	31	0
Positive	BCG+others	10	27	34	29

Table 4
Fractions of choices in non-dominated strategies intervals.

Interval	BCG−others	BCG+others	BCG−	BCG+
I_0	1	1	1	1
I_1	0.954545455	0.89375	0.833333333	0.625
I_2	0.920454545	0.84375	0.819444444	0.5625
I_3	0.886363636	0.68125	0.694444444	0.3125
I_4	0.806818182	0.64375	0.583333333	0.3125
I_5	0.698863636	0.34375	0.402777778	0.09375

chooses a number which is a best reply of his estimated distribution of depths of reasoning in the population. We estimated the average depth of reasoning for each treatment (assuming a Poisson distribution, see Appendix A.3) in order to ensure that subjects were drawn from the same population, i.e. they were equally skilled under both conditions. The estimated average depths of reasoning are 1.55 for BCG−, 2.20 for BCG−others, 1.48 for BCG+ and 2.56 for BCG+others. Table 2 reports these values together with the average winning number in each game. There is no significant difference in the depth of reasoning between BCG+ and BCG−, and between BCG+others and BCG−others (p value>0.7). Distributions are similar within a strategic design (between BCG+ and BCG−, and between BCG+others and BCG−others). We conclude that BCG+ and BCG− subjects are drawn from the same population; and equivalently for BCG+others and BCG−others subjects.

The depth of reasoning is larger in the non-strategic treatments (BCG−others and BCG+others) than in the corresponding standard treatments (BCG− and BCG+). Subjects seem to have a deeper guessing ability when the task is to guess the others mean (p *value*<0.0009). Table 3 reports the estimated distributions of reasoning depths for all treatments. A player with a k depth of reasoning performs k steps of introspection and is defined as a k-step player. Our results report a population with 0-, 1- and 2-steps players in standard designs and a population with up to 3-steps players in the non-strategic design. This is consistent with our condition and consistent with Camerer's findings (Camerer, 2003) about a natural depth of reasoning of about two steps of introspection.

While the CH model rejects the hypothesis that the average depth of reasoning is larger under negative feedback than under positive feedback, Result 1 showed that subjects are closer to the equilibrium value under negative feedback. We therefore need to find another explanation. We first establish (Result 3) that under negative feedback, subjects "use" more intensively higher order intervals. We measure the frequency of chosen numbers in non-dominated interval for different levels of the eductive process.

Result 3. More subjects choose numbers in higher-order intervals under negative feedback than under positive feedback, independently of the starting point (x) used to compute the sequence of non-dominated intervals.

Support for Result 3: Table 4 shows the percentage of choices in each non-dominated strategies interval for the first five steps of reasoning, starting at value $x=100$. Table A1 (Appendix A.3) provides the same data for other values of x (90,80,…,10).

Clearly, all intervals are more intensively "used", and the higher the interval order, the higher the difference in reported percentages in this interval in the BCG− and the BCG−others than in BCG+ and BCG+others, respectively. We test for equal frequency of chosen numbers in each interval under positive and negative feedback by a one-sided frequency test. The null hypothesis is rejected most of the time a the 5% level in favour of the hypothesis that more numbers are chosen in each interval under negative feedback, for all values of x.[5]

[5] The hypothesis of equal frequency is rejected only four times (out of 45 comparisons) for "non-strategic" treatments and seven times (out of 45 comparisons) for "strategic treatments".

5. Discussion

Our main finding is that winning numbers are closer to the predicted outcome under BCG− than under BCG+, although we cannot reject the hypothesis that the average depth of reasoning is equal in both games. These results are partly in line with the findings of Heemeijer et al. (2008) and Fehr and Tyran (2008). Before we discuss our own results, let us briefly recall the explanations provided in these two papers. Although the two experiments use different settings and investigate different issues, they both compared a simple price guessing game-which admits a unique equilibrium[6]-under two feedback rules: strategic substitutes vs. strategic complements. Under strategic substitutes (negative feedback), Heemeijer et al. (2008) found prices to be relatively stable, often moving closely towards the equilibrium level. In contrast, under strategic complements (positive feedback), they observed oscillatory price movements with persistent deviations from the equilibrium value. They attribute the difference to a stronger tendency towards *trend following behaviour* under positive feedback: if many players follow such a strategy it is more profitable for a player to adopt the same behaviour, which reinforces the trend. Exactly the opposite is true in a negative expectations feedback environment, with the consequence of weakening the trend. Therefore, trend following behaviour is less likely to survive in markets with a negative feedback structure, because of a strong incentive to adopt contrarian behaviour. A similar interpretation is provided in Fehr and Tyran (2008), leading them to the conclusion that errors are more salient and costly under negative feedback: "the higher saliency of the error means that detecting the error is less cognitively costly; the higher economic cost associated with the error implies that the gains from avoiding the error are higher". They attribute the higher saliency and higher cost of the error under strategic substitutes to the fact that a rational player has a strong incentive to choose an action that is "far away" from that of an adaptive player, i.e. makes a larger payoff gain from playing rationally. In contrast, under strategic complementarity, moving away from the action of an adaptive player does not lead to a substantially larger payoff for a rational player, making the cost of error less salient.

While the interpretations offered by Heemeijer et al. (2008) and Fehr and Tyran (2008) are compelling, they do not translate easily to our data. The reason is that our experiment did not involve any explicit dynamic process, which would help subjects to adjust their actions over time. Intuitively, in the BCG subjects rely on an introspective process, whereby they perform virtual steps in notional time, rather than explicit steps in real time. Surprisingly, the outcome of this unobservable introspection process parallels the findings of the explicit dynamic process of the price guessing game of Heemeijer et al. (2008) and Fehr and Tyran (2008). Further evidence for this is given in our related papes (Sutan and Willinger, 2005, 2006) in which we report data from a 10 times repeated BCG game, with the same interior equilibrium (60) under positive and negative feedback. In contrast to the one-shot game, the repeated game allows subjects to adjust their current strategy to past winning numbers, and therefore learning plays a crucial role. Although the learning issue is beyond the scope of this paper, there is one important finding that is relevant to our discussion. Except for period 1, we found no significant difference in the average deviation to the equilibrium value, between positive and negative feedback. When subjects can adjust their current strategy with respect to past winning numbers, on average they tend to reduce the deviation with respect to the equilibrium value equally well under both feedback conditions. The average deviation with respect to the equilibrium value was measured by the mean squared deviation and the mean absolute value deviation. Taking either of these two measures, there is a significantly larger deviation in period 1 under positive feedback compared to negative feedback, but no such difference for all subsequent periods. This is a further reason to conjecture that subjects think differently in the negative feedback environment. What really seems to matter is the unobservable cognitive process that leads them to the selection of a number.

Could it be then, that subjects perform a deeper reasoning under negative feedback than under positive feedback? According to our Result 2 (Section 4), the answer would be "no". The estimated average depths of reasoning based on the CH model are remarkably similar for BCG+ and BCG−, and the two distributions coincide almost perfectly. There is some difference between BCG+others and BCG−others, but actually in favour of a deeper reasoning under positive feedback. However, according to Result 3 (Section 4), the answer to the previous question would be "yes"!. Subjects might perform more steps of reasoning for other reasons than those underlying CH. Below, we suggest that such performance is likely to be driven by a higher value of expected information under negative feedback.

Following Gabaix et al. (2006) and Gabaix and Laibson (2005) directed cognition theory, we may consider the introspective process underlying one-shot BCG as a bounded rational process: at each stage a player decides myopically to perform an additional step of reasoning by trading off the cognitive cost of the additional step with its expected benefit. This process differs from an optimal cognition process which assumes that players equate marginal cost of thinking to marginal benefit, by taking into account (non-myopically) all potential steps of reasoning. Assuming a step-by-step evaluation, we suggest that for equal cognitive cost under positive and negative feedback, the latter generates more useful information (expected benefit) for locating the equilibrium. The higher value of information under negative feedback results from the two-sided elimination of dominated strategies. In Section 2 we showed that the sequences of eliminated intervals for BCG+ and BCG− coincide either in even periods or in odd periods, depending on whether BCG+ is considered or its' complementary process. Starting from any point x of the strategy interval, players always eliminates more weakly

[6] The experiment of Fehr and Tyran (2008) involves heterogeneous payoff functions, which implies that their equilibrium prediction is an average, while individual equilibrium expectations are slightly different according to types.

dominated numbers under negative feedback than under positive feedback. Therefore, each step of reasoning generates more useful information for locating the winning number under negative feedback. In Result 3 we showed that, for a selection of values of x covering the whole range of possible values, the frequency of chosen numbers is most of the time larger for any depth of reasoning, under negative feedback. This result suggests that subjects' choices are compatible with the value of information interpretation. Under negative feedback, there is an incentive for deeper thinking, because it is more valuable than under positive feedback, all things equal.

Additionally, there is a reason to believe that the cost of locating the equilibrium point is lower under negative feedback. Recent findings in cognitive psychology established that numbers are perceived on a mental scale with a left-to-right orientation (Dehaene, 1993). Starting the eductive process at any number x, under positive feedback the winning number is never scanned, while under negative feedback it is scanned at each step of the reasoning process. Therefore, the mental cost for locating the equilibrium value might be lower under negative feedback.

6. Conclusion

In this paper we investigated if the type of feedback-or strategic environment-underlying the reasoning process could be a reason in subjects' guessing success. We compared two possible feedback rules: positive and negative. Under positive feedback, subjects' actions are strategic complements. Assuming eductive reasoning, the iterated elimination of dominated strategies corresponds to a process where strategies are eliminated from one side of the equilibrium point. The standard BCG with an equilibrium point at zero is an illustration of this process, but BCG with positive feedback can also have interior equilibria as well, depending on how the winning number is defined. Under negative feedback, subjects' actions are strategic substitutes. In this case, eductive reasoning implies the elimination of strategies from both sides of the equilibrium point by alternately eliminating numbers below and above the equilibrium point. Therefore, under negative feedback, the equilibrium point is necessarily within the limits of the strategy space.

Our main finding is that under negative feedback winning numbers are much closer to the equilibrium than under positive feedback. The average depth of reasoning, estimated according for instance to the CH model, is not different between the two feedback rules. In the CH model, the main assumption is made on the fact that players choose numbers which best reply to their estimated distribution of reasoning depths in the population. However, we show that the data is compatible with the conjecture that subjects have an incentive towards deeper reasoning, because each additional step of thinking is more valuable under negative feedback.

Our results are closely related to the recent experimental findings by Heemeijer et al. (2008) and Fehr and Tyran (2008), for price guessing games when guesses are strategic substitutes or strategic complements. While our game is one-shot, the outcome of the unobservable introspective process of guessing the winning number parallels the findings of the price guessing games. Under strategic substitutes price expectations converge closer to the equilibrium value than strategic complements.

While the CH model does not account for the difference between the negative and the positive feedback environments, such difference might be compatible with a generalized notion of eductive reasoning. As pointed out in footnote 1 (quoting Binmore, 1987), eductive reasoning is just one ingredient of a more general decision process which corresponds to the entire reasoning activity that intervenes between the receipt of a decision stimulus and the ultimate decision, including the manner in which the agent forms the beliefs on which the decision will be based. By taking into account the costs and benefits of thinking, eductive reasoning is a generalized theory of step-by-step reasoning, and it can account for the observed differences between the two feedback rules.

Appendix A

A.1. *Example of instructions*

Welcome!

The goal of this experiment is to study how individuals make decisions. The instructions are simple and if you follow them carefully you will receive a certain amount of money in cash by the end of the experiment. Payments will be made confidentially, so no one will receive information about the earnings of the other participants. You can ask a question at any time by raising your hand first. Apart from these questions it is strictly forbidden to talk among participants. Talking may result in immediate expulsion from the experiment.

There are four groups of eight people each in this room. Therefore, in your group, there are eight participants, including you.

In this experiment, you have to choose a number. All members in your group will have to choose numbers. Among all participants, it will be a winner and he will win 8 Euros. If you want to be a winner of the group and to earn the prize, the number that you choose has to be the closest possible to a target determined by: $100-2/3$ (mean of all chosen numbers in your group).

Example: If you choose eight and the others seven participants in your group choose 0, the target is $100-2/3(0+0+0+0+0+0+0+8)/8 = 100-2/3*1 = 99.33$. In this case, you are the winner, because 8 is closer to 99.33 than 0 to 99.33.

If there are several winners, the 8 euros will be split among them. Good luck!

A.2. Best reply functions

We Compute the magnitude of the slope of the best reply function for the case $p = 2/3$. $h = 100$, $l = 0$, $c = 30$. When the winning number in the BCG− and in the BCG+ is related to the mean chosen by all (in the BCG−100−2/3mean, in the BCG+2/3(30+mean), player i's best reply to the average chosen by the others ($\overline{x_{-i}}$) is to choose:

$$x_i(\text{BCG}-) = \frac{n}{n+p} - \frac{p(n-1)}{n+p}\overline{x_{-i}} \text{ in the BCG} - \text{but,}$$

$$x_i(\text{BCG}-) = 100\frac{np}{n-p} - \frac{p(n-1)}{n-p}\overline{x_{-i}} \text{ in the BCG} - \text{but.}$$

While the equilibrium is the same (at 60) in both treatments, the absolute slope of the best reply function is different in the two treatments:

$$\frac{\partial x_i}{\partial \overline{x_{-i}}}(\text{BCG}-) = -\frac{p(n-1)}{n+p} = -\frac{7}{13}$$

but

$$\frac{\partial x_i}{\partial \overline{x_{-i}}}(\text{BCG}+) = \frac{p(n-1)}{n-p} = \frac{7}{11}.$$

As a consequence, the range of dominated strategies is different: from (approximately) 92.3 (for $\overline{x_{-i}} = 0$) to 38.5 (for $\overline{x_{-i}} = 100$) in the BCG−, but from 21.8 to 85.5 in the BCG+ (for $p = 2/3$). Hence, the range of non-dominated choices is wider in BCG+ than in BCG− (about 63.6 vs. 53.8) which means that BCG+ is "more difficult".

Under the assumption that the target number is $\overline{x_{-i}}$ (the mean of the numbers chosen by all players except i) rather than $\bar{x}$, the absolute value of the slope and the range of non-dominated strategies are the same (p and 66.6 for $p = 2/3$).

A.3. The CH model (Camerer et al., 2004)

Level-0 players choose randomly, with equal probability, any number between 0 and 100. A level-k' player believes that he faces a population of players of lower level, i.e. players of level $k = 0$ to level $k = k'-1$. Furthermore, the CH model assumes that the proportion of level k players in the population is a decreasing function of k. Assume that the beliefs of level k-players about the proportions of level k'-players, $g_k(k')$, is the normalized true distribution ($g_k(k') = f(k')/\sum_{l=0}^{k-1} f(l)$, for $h<k$). Level k-players chose a number which is a best reply to the estimated average number chosen by the other players, computed according to their beliefs. Following Camerer and et al. (2004), we assume that deeper reasoning is increasingly rare due to working memory constraints and doubts about the rationality of others. This is captured by letting $f(k)/f(k-1)$ be proportional to $1/k$ which implies that $f(k) = e^{-\tau}\tau^k/k!$, the Poisson distribution, where τ is the mean and variance of the number of reasoning steps. Camerer and et al. (2004) found for BCG+ that τ lies between 1 and 2, which means that, in the one-shot game, players do not compute more than two steps of reasoning. Table A1 provides the same data for other values of x (90,80,…,10).

Table A1
Frequency distribution of numbers for the five first intervals (I_1–I_5) of the eductive reasoning process for different values of x.

Value of x	Interval	BCG−others	BCG+others	BCG−	BCG+
100	I0	1	1	1	1
	I1	0.954545455	0.89375	0.833333333	0.625
	I2	0.920454545	0.84375	0.819444444	0.5625
	I3	0.886363636	0.68125	0.694444444	0.3125
	I4	0.806818182	0.64375	0.583333333	0.3125
	I5	0.698863636	0.34375	0.402777778	0.09375
50	I0	1	1	1	1
	I1	0.784090909	0.6375	0.583333333	0.3125
	I2	0.676136364	0.34375	0.402777778	0.09375
	I3	0.409090909	0.28125	0.166666667	0.03125
	I4	0.340909091	0.21875	0.111111111	0
	I5	0.318181818	0.16875	**0.097222222**	**0**

Table A1 (*continued*)

Value of *x*	Interval	BCG−others	BCG+others	BCG−	BCG+
90	I0	1	1	1	1
	I1	0.931818182	0.8125	0.777777778	0.5
	I2	0.869318182	0.76875	0.722222222	0.40625
	I3	0.778409091	0.575	0.486111111	0.1875
	I4	**0.625**	**0.54375**	0.361111111	0.1875
	I5	0.551136364	0.30625	0.305555556	0.09375
	I0	1	1	1	1
80	I1	0.909090909	0.70625	0.708333333	0.34375
	I2	0.823863636	0.675	0.583333333	0.3125
	I3	0.715909091	0.36875	0.402777778	0.09375
	I4	0.4375	0.325	0.194444444	0.03125
	I5	0.369318182	0.24375	**0.138888889**	**0.03125**
	I0	1	1	1	1
70	I1	0.755681818	0.5375	0.472222222	0.1875
	I2	**0.488636364**	**0.46875**	**0.25**	**0.15625**
	I3	0.409090909	0.24375	0.194444444	0.0625
	I4	0.323863636	0.16875	**0.097222222**	**0.03125**
	I5	0.318181818	0.16875	**0.097222222**	**0**
	I0	1	1	1	1
40	I1	0.869318182	0.76875	0.722222222	0.40625
	I2	0.778409091	0.575	0.486111111	0.1875
	I3	**0.625**	**0.54375**	0.361111111	0.1875
	I4	0.551136364	0.30625	0.305555556	0.09375
	I5	0.375	0.24375	**0.125**	**0.03125**
	I0	1	1	1	1
30	I1	0.943181818	0.86875	0.833333333	0.59375
	I2	0.909090909	0.70625	0.708333333	0.34375
	I3	0.8125	0.66875	0.583333333	0.3125
	I4	0.704545455	0.3625	0.402777778	0.09375
	I5	0.4375	0.325	0.194444444	0.03125
	I0	1	1	1	1
20	I1	0.965909091	0.9375	0.930555556	0.8125
	I2	0.931818182	0.775	0.777777778	0.4375
	I3	0.869318182	0.74375	0.722222222	0.40625
	I4	0.767045455	0.55625	0.486111111	0.1875
	I5	**0.613636364**	**0.525**	0.361111111	0.1875
10	I0	1	1	1	1
	I1	0.982954545	0.96875	**0.944444444**	**0.9375**
	I2	0.931818182	0.8375	0.777777778	0.5625
	I3	0.880681818	0.79375	0.75	0.46875
	I4	0.857954545	0.66875	0.680555556	0.3125
	I5	0.784090909	0.6375	0.583333333	0.3125

Bold values do not differ significantly correspond to non-significant differences (one sided frequency test, 5% significance level). N.B. all differences are significant at the 10% level.

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